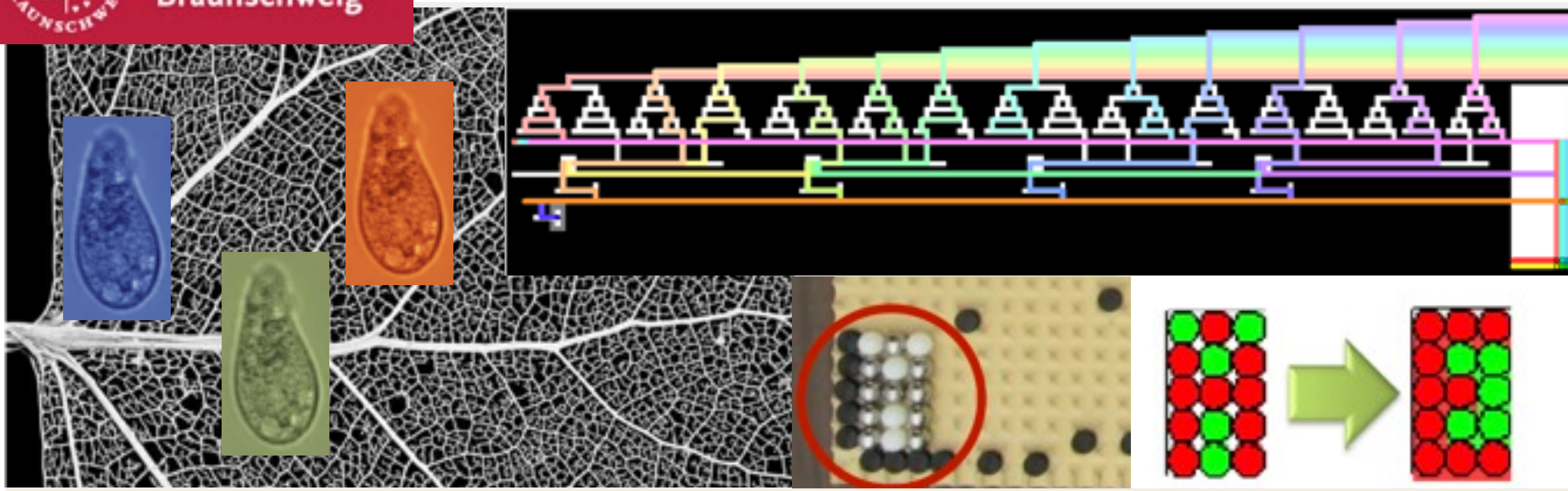
Technische Universität Braunschweig

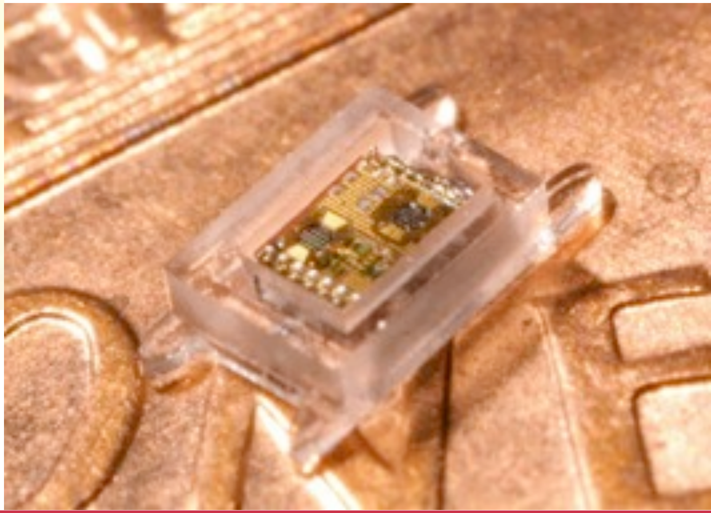


Controlling Distributed Swarms with only Global Signals

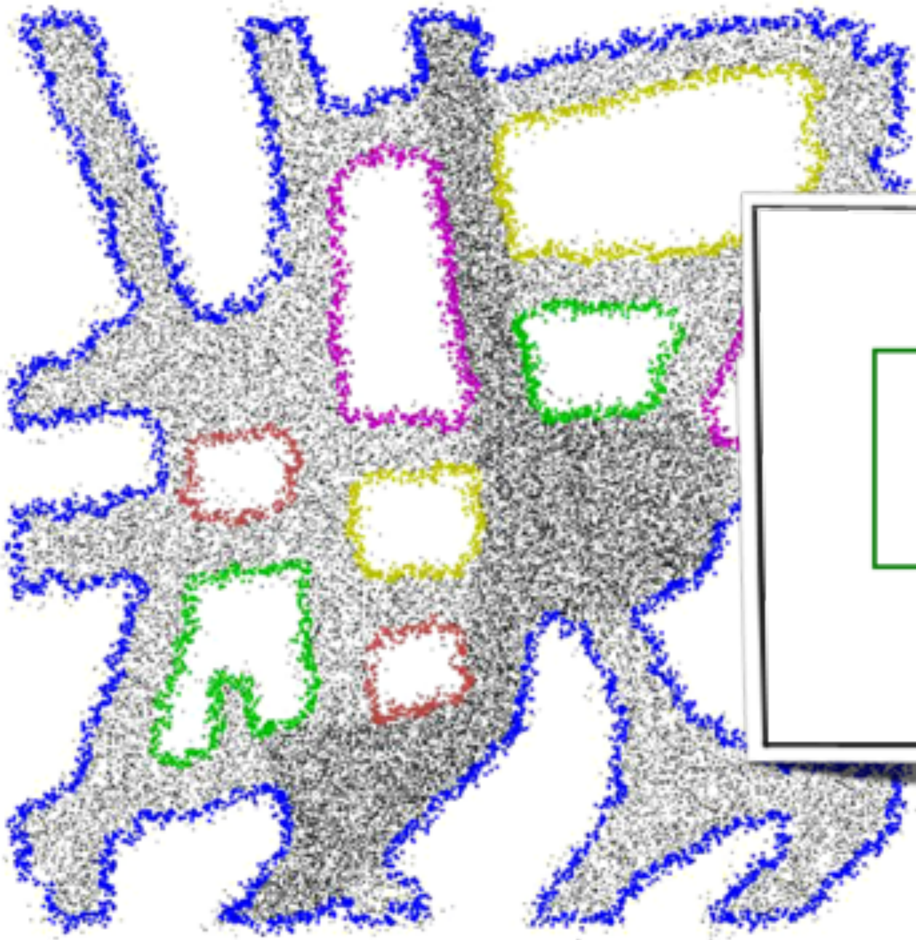
Aaron Becker, Erik D. Demaine
Sándor P. Fekete

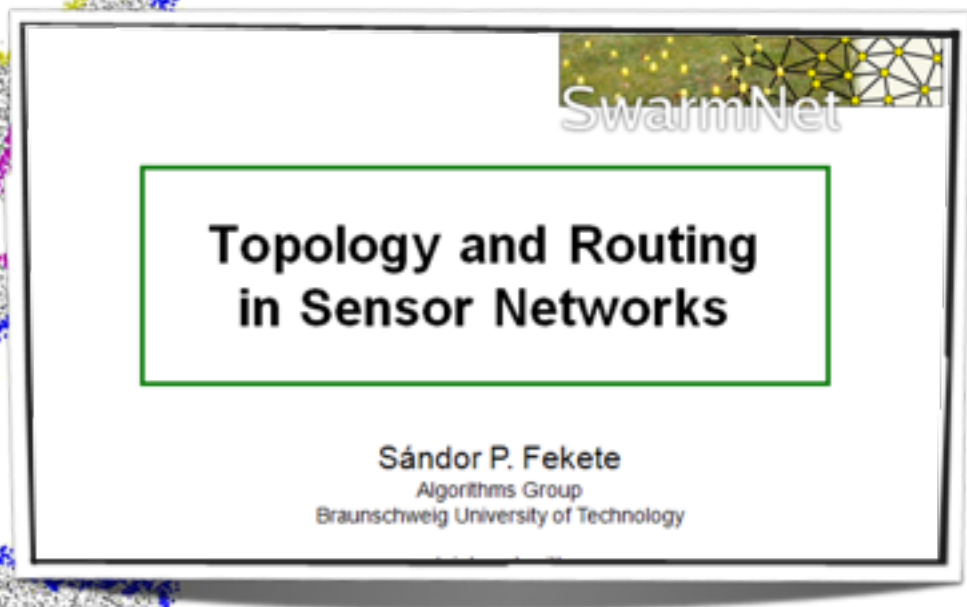
Part I: Processors and Mobile Objects

Processors

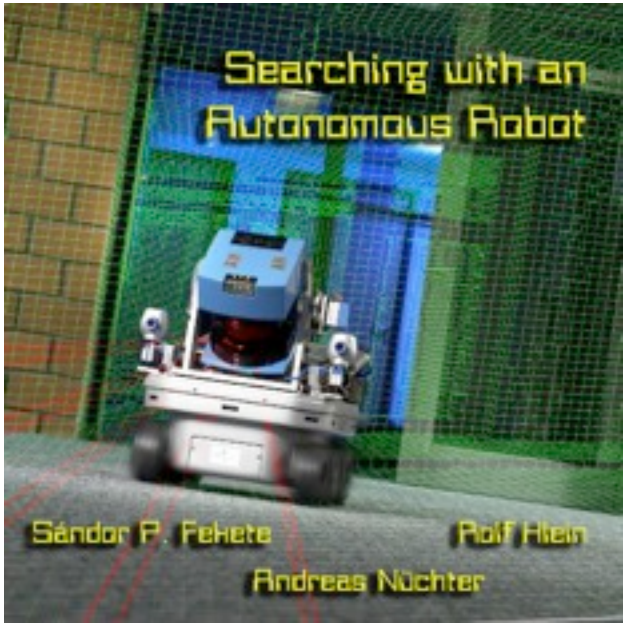


“Smart Dust”

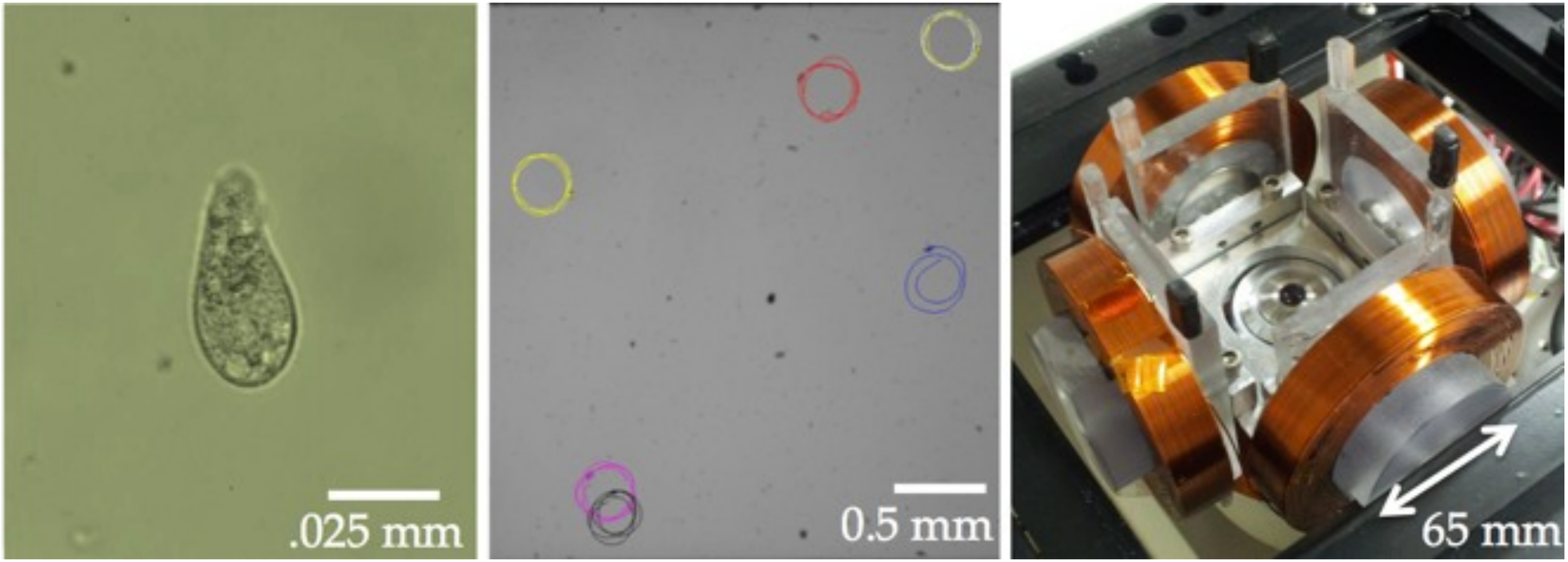



SwarmNet
Topology and Routing
in Sensor Networks
Sándor P. Fekete
Algorithms Group
Braunschweig University of Technology

Mobile Objects and Robots



Moving Small Objects

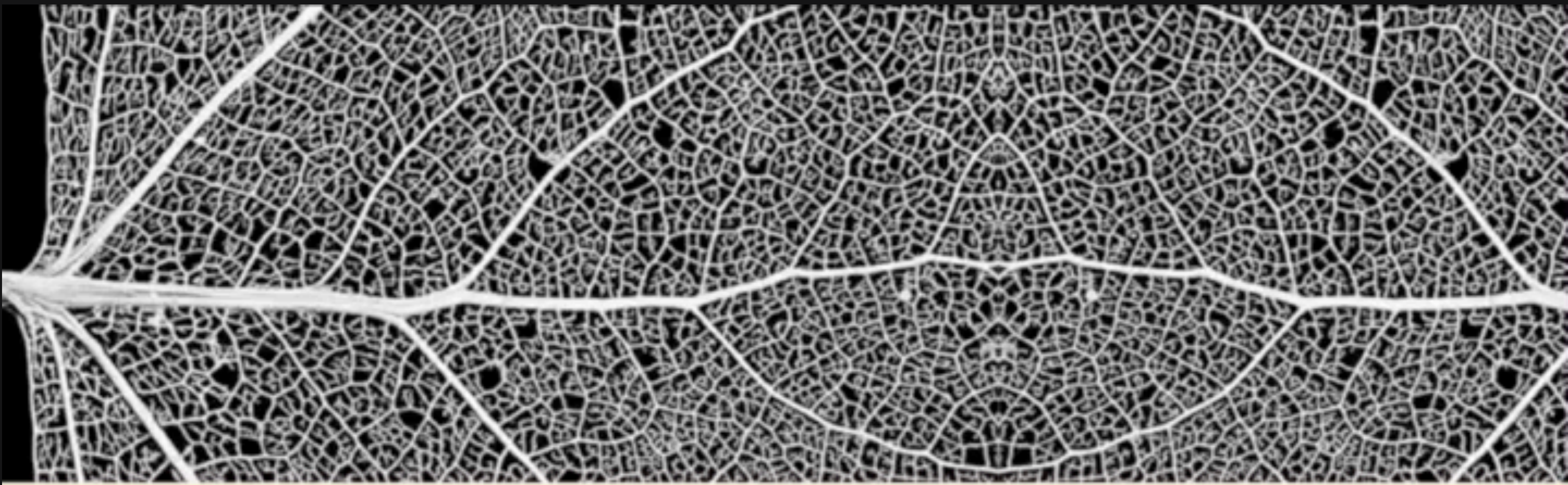


Tetrahymena pyriformis

This Talk

- Massive particle swarms
- Global control, not individual motion
    - *We show hardness for given, external obstacles*
    - *We establish positive results for designed, additional obstacles*
- Work in progress, combining theory and practice

Video



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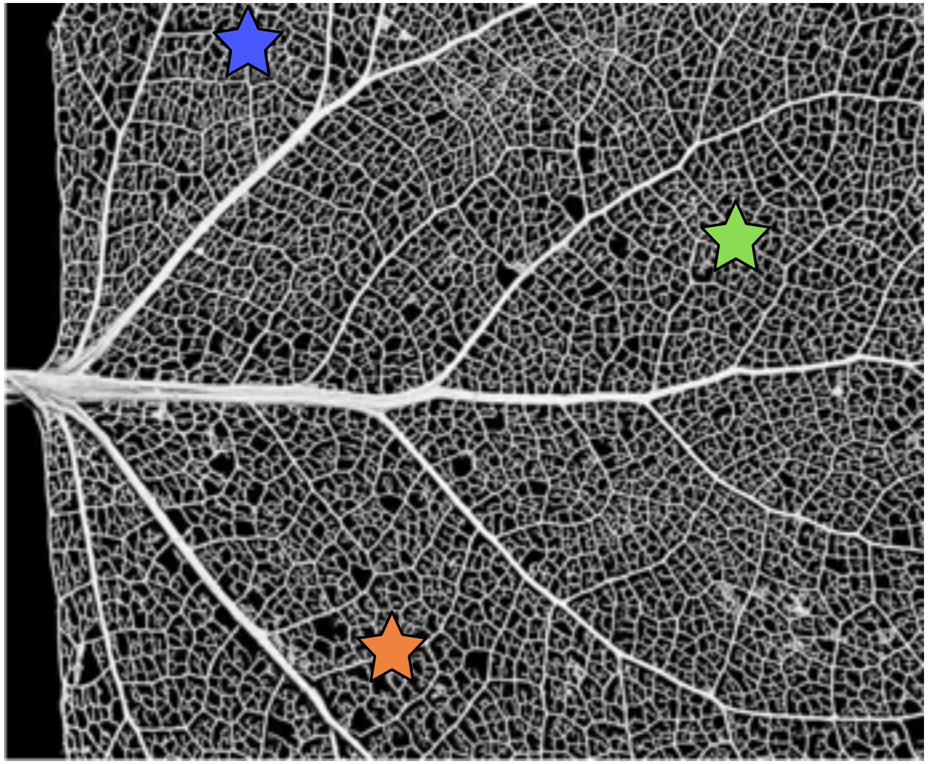
Part II: Why Obstacles Are a Nuisance

Obstacles as Opponents

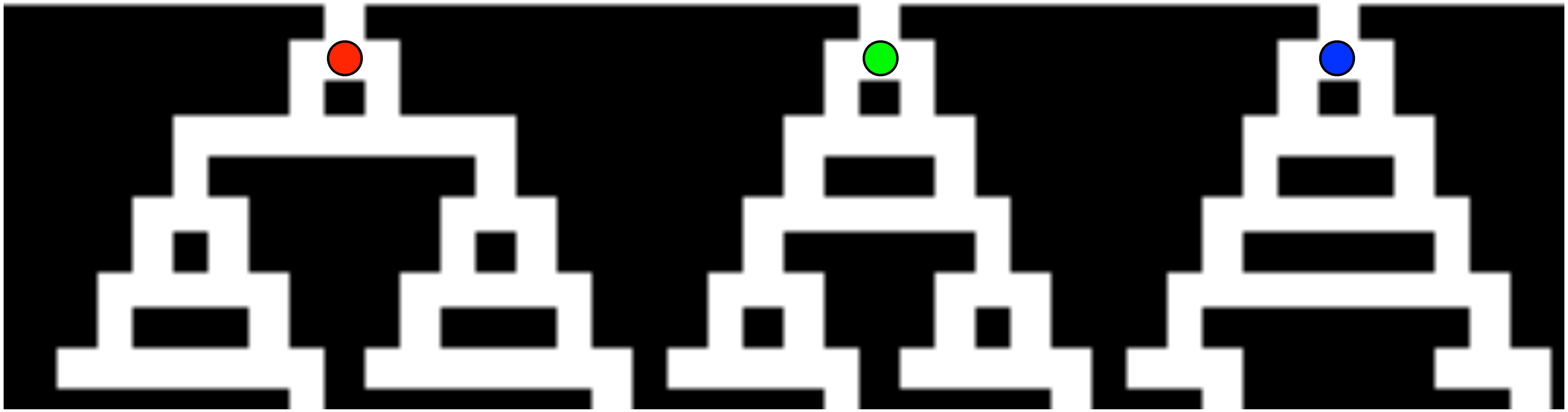
- Targets may not be easy to reach.
- Motion planning gets quite tricky in parallel.



Cottonwood leaf vascular network

Complexity: Binary Variables

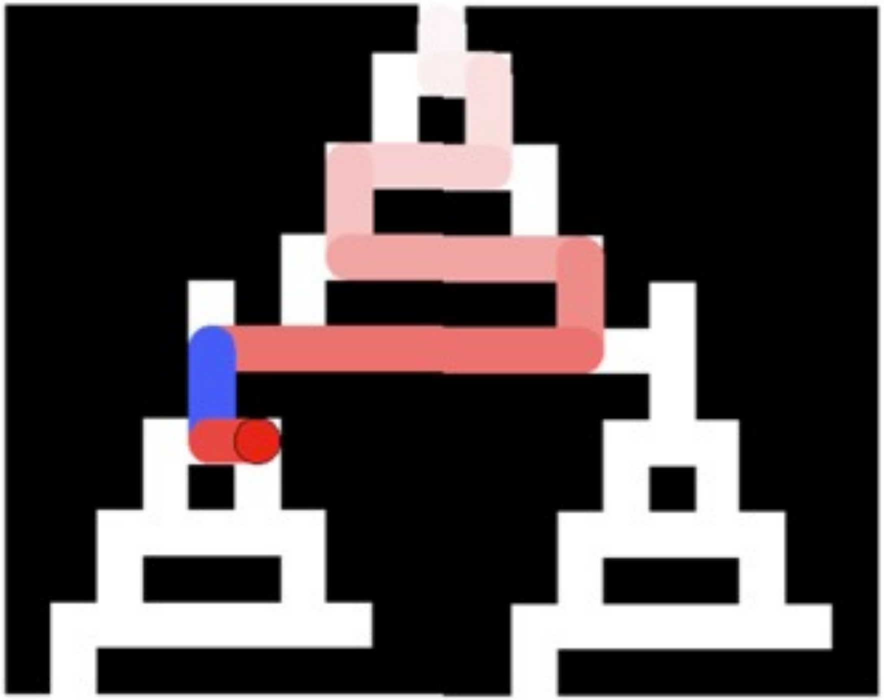
Choice: left or right?
Independent choices?!



x_2 x_3 x_4

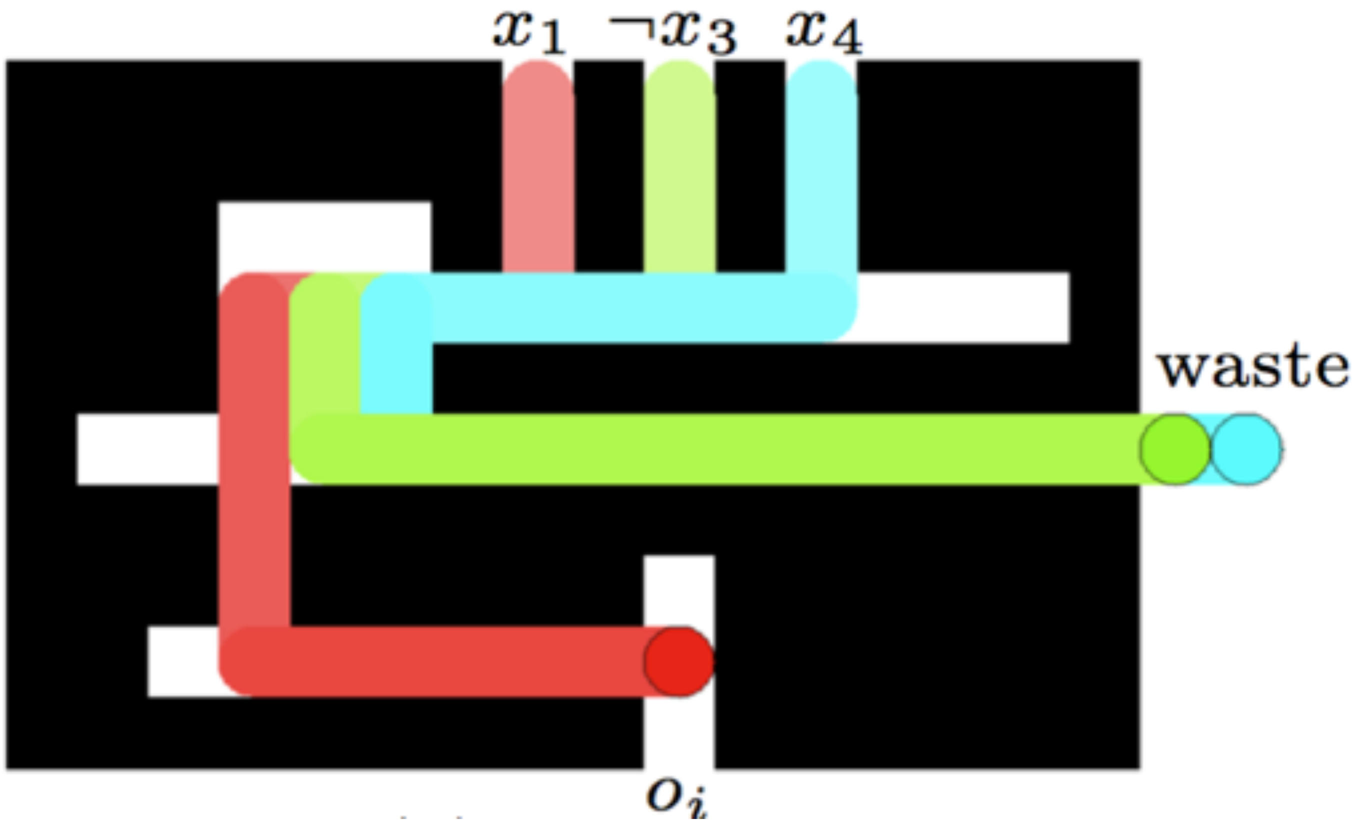
Choice only matters when it is a variable's "turn"!

Complexity: Binary Variables

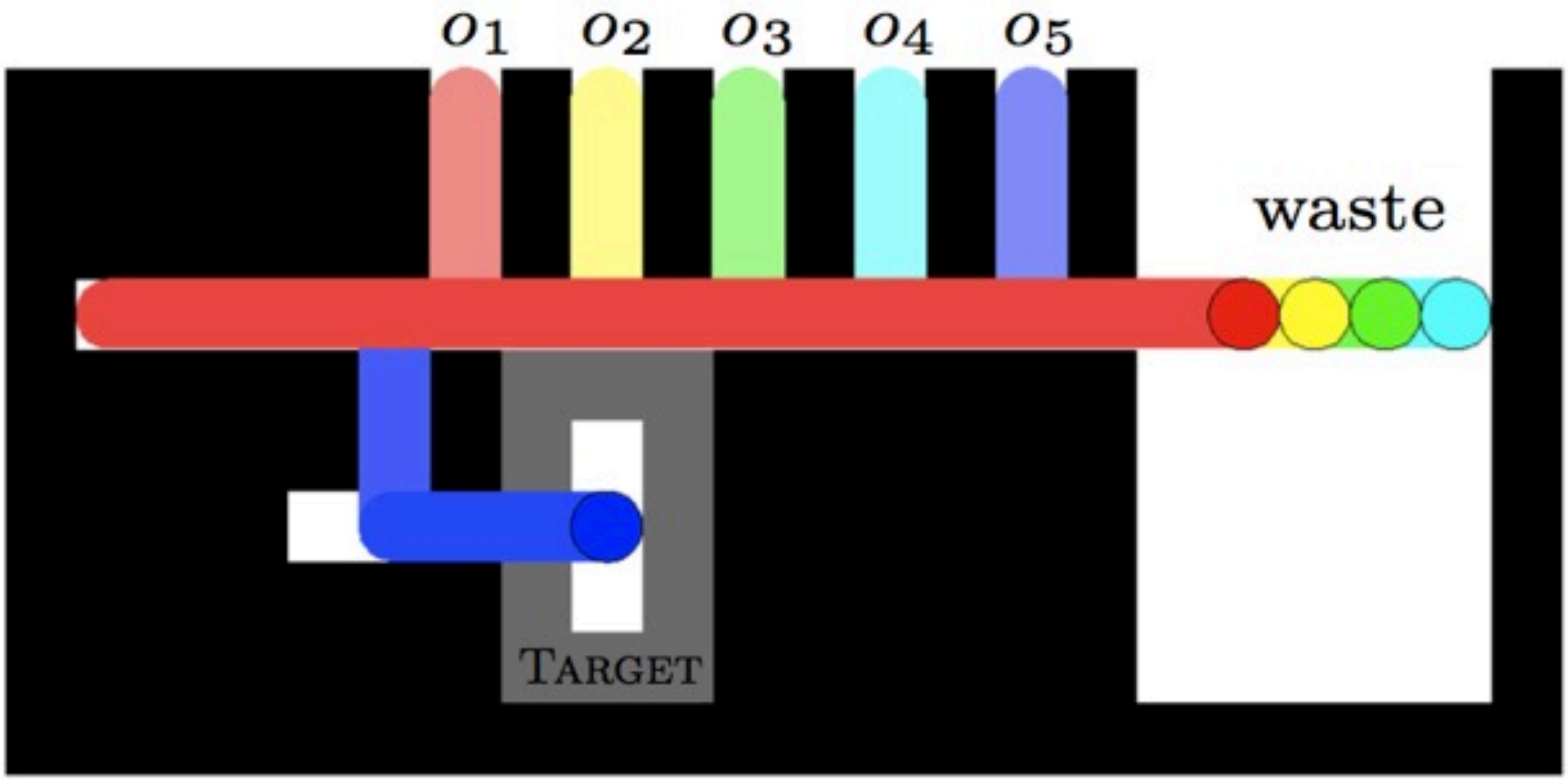


Minor detail: Avoid reversible choices!

Complexity: Clauses

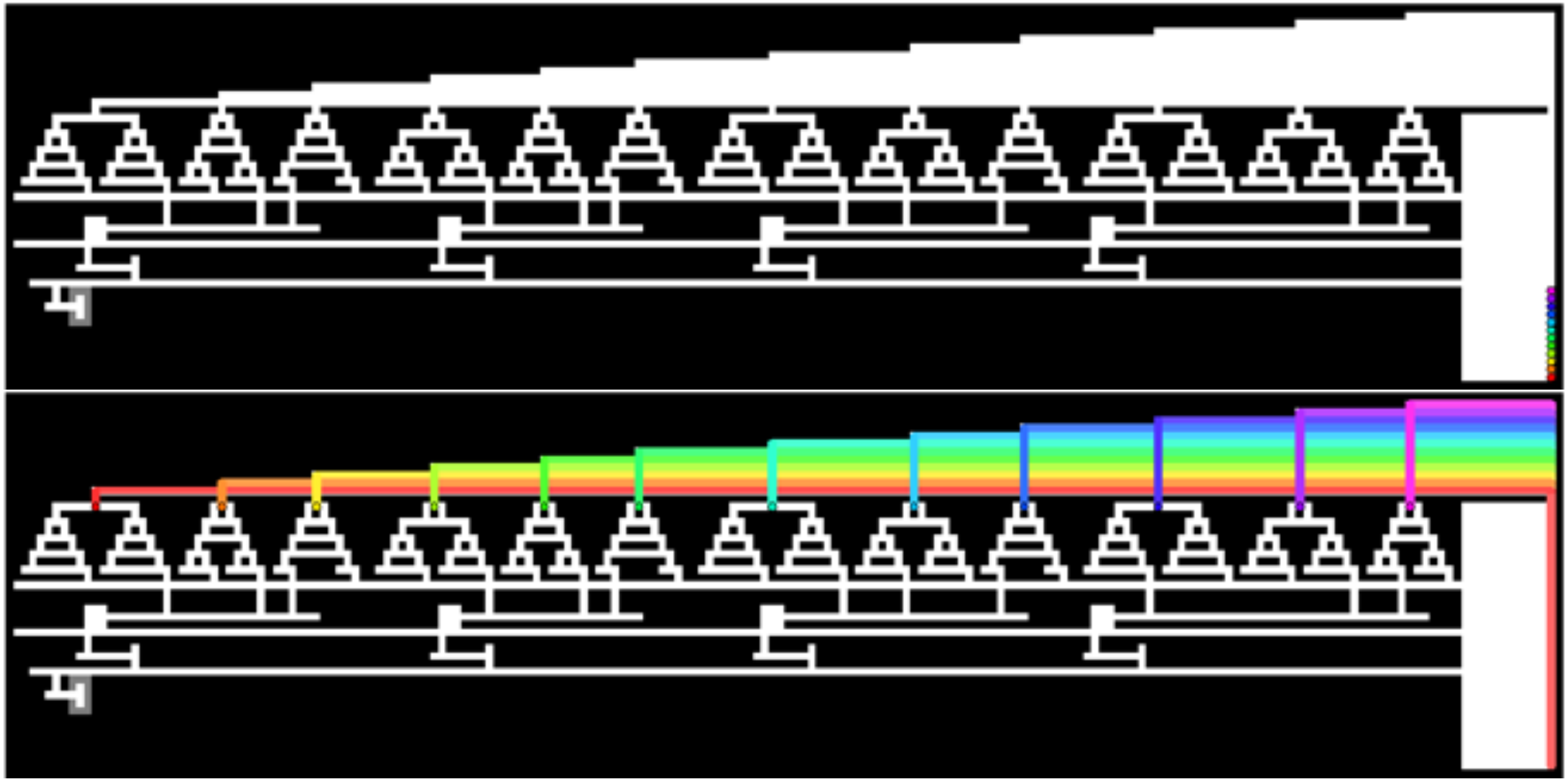


Complexity: Truth Checking



Complexity: Overall Construction

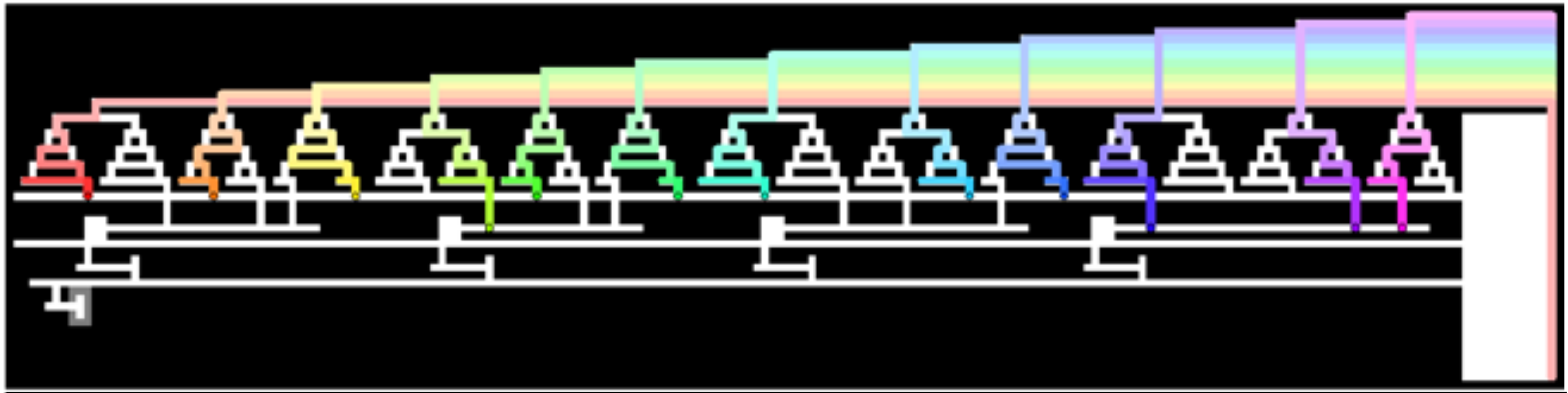
$$(\neg x_1 \vee \neg x_3 \vee x_4) \wedge (\neg x_2 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee x_2 \vee x_4) \wedge (x_1 \vee \neg x_2 \vee x_3)$$

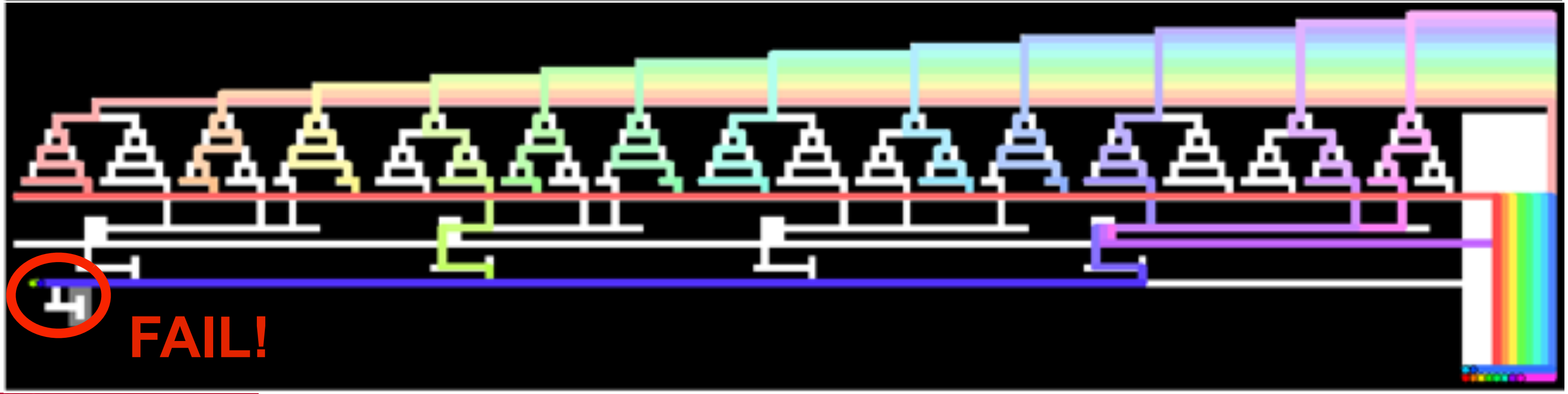


Complexity: Overall Construction

$(\neg x_1 \vee \neg x_3 \vee x_4) \wedge (\neg x_2 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee x_2 \vee x_4) \wedge (x_1 \vee \neg x_2 \vee x_3)$

$x_1 = 1, x_2 = 0, x_3 = 1, x_4 = 0$

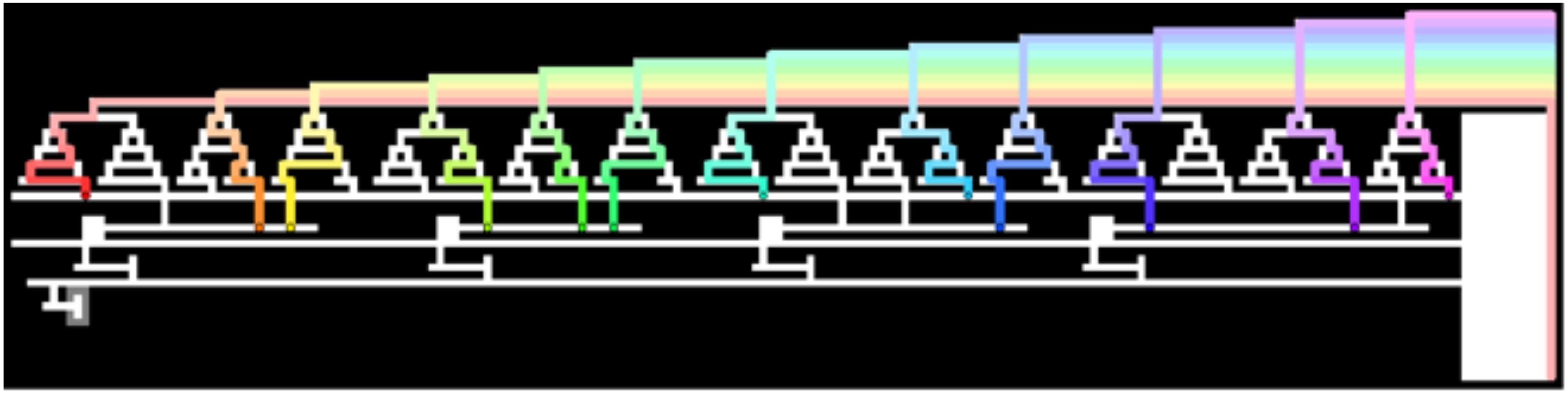


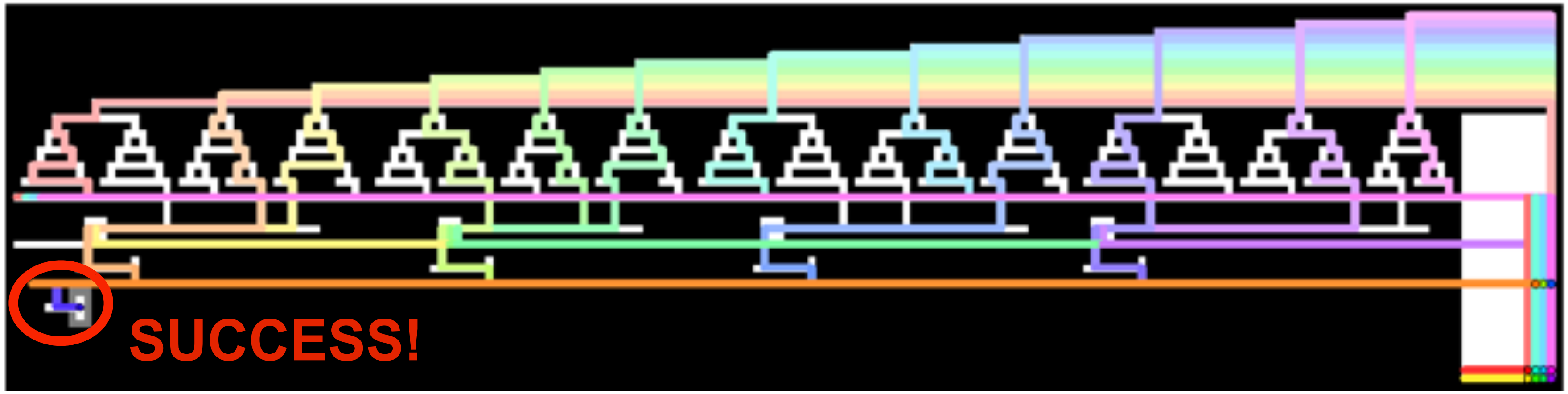


Complexity: Overall Construction

$(\neg x_1 \vee \neg x_3 \vee x_4) \wedge (\neg x_2 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee x_2 \vee x_4) \wedge (x_1 \vee \neg x_2 \vee x_3)$

$x_1 = 1, x_2 = 0, x_3 = 0, x_4 = 1$





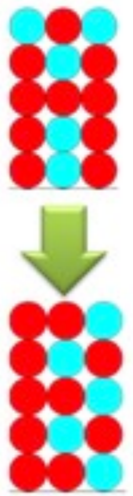
Complexity: Summary

Theorem 1. GLOBALCONTROL-MANYPARTICLES *is NP-hard: given an initial configuration of movable particles and fixed obstacles, it is NP-hard to decide whether any particle can be moved to a specified location.*

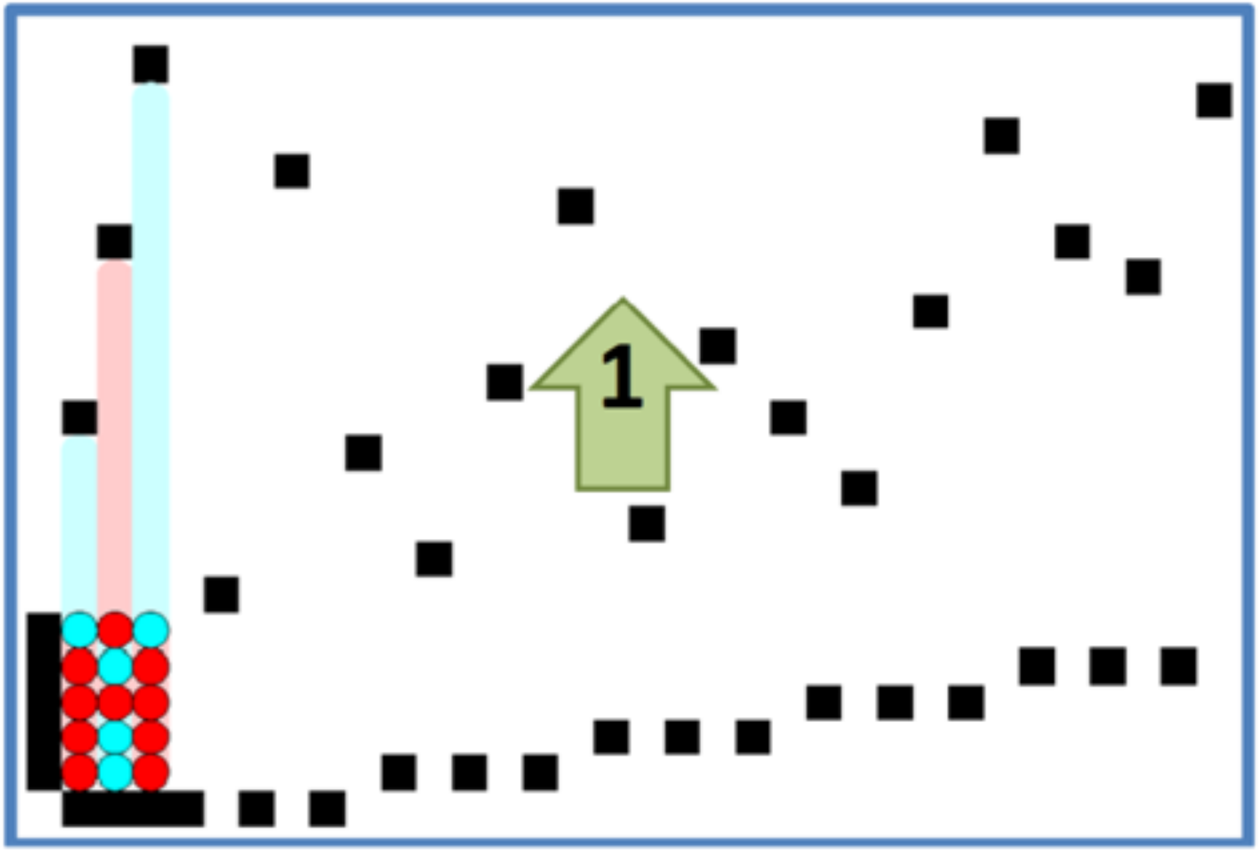
Part III: Why Obstacles Are a Blessing

Life without Obstacles

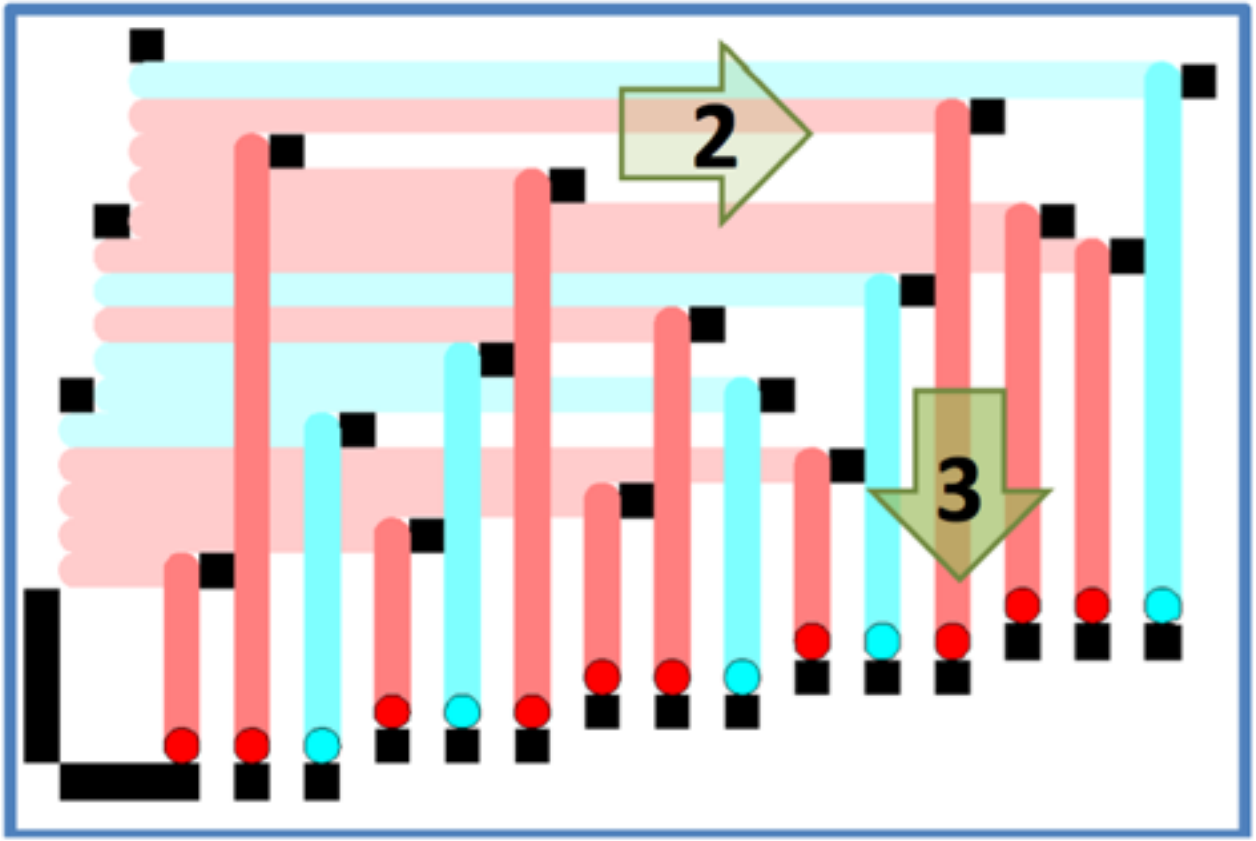
Lack of obstacles can be harmful!



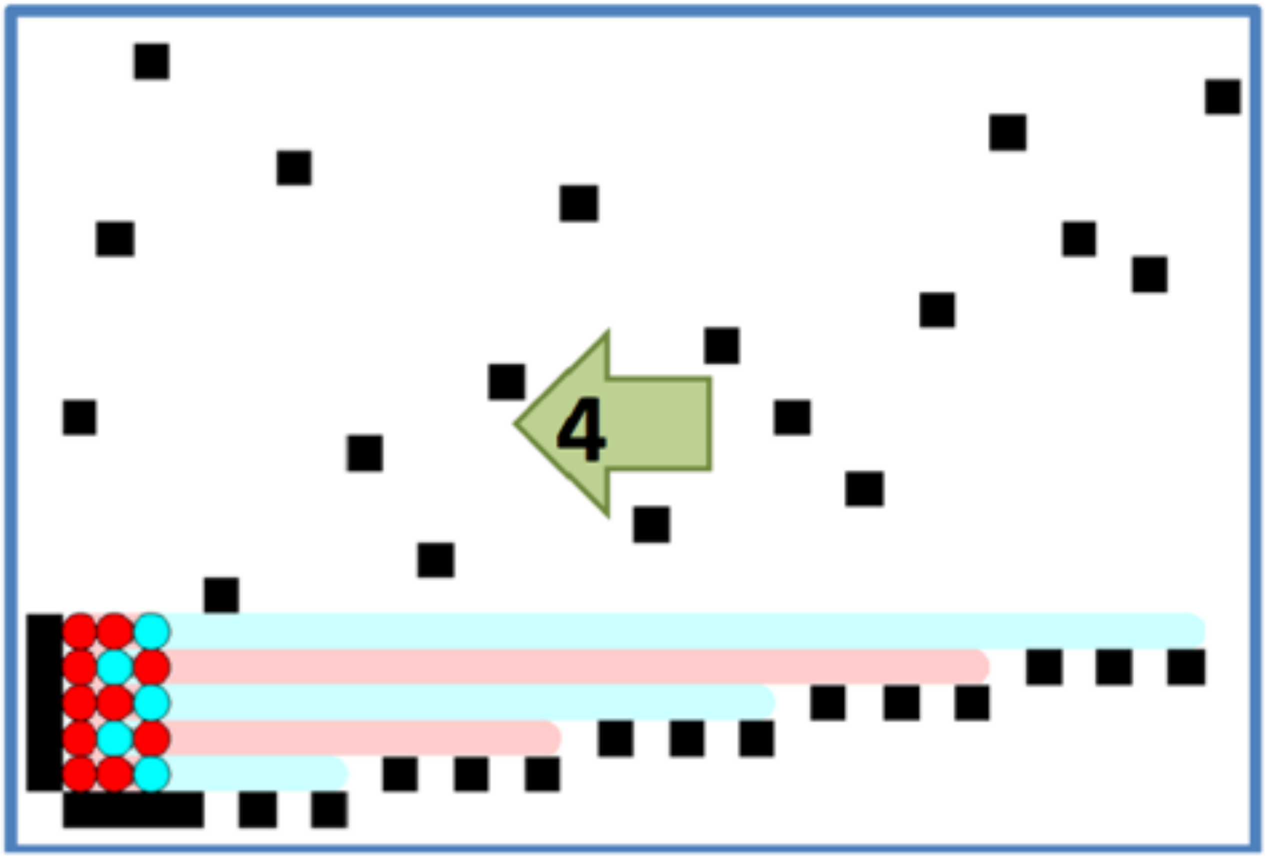
How Obstacles Can Be Helpful



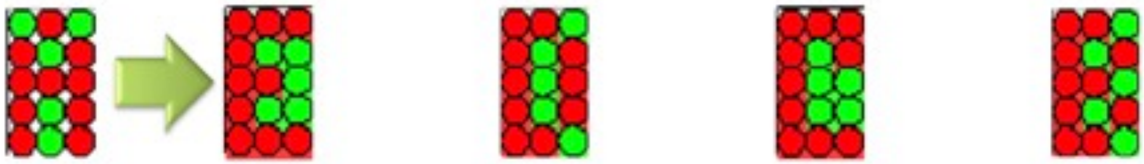
How Obstacles Can Be Helpful

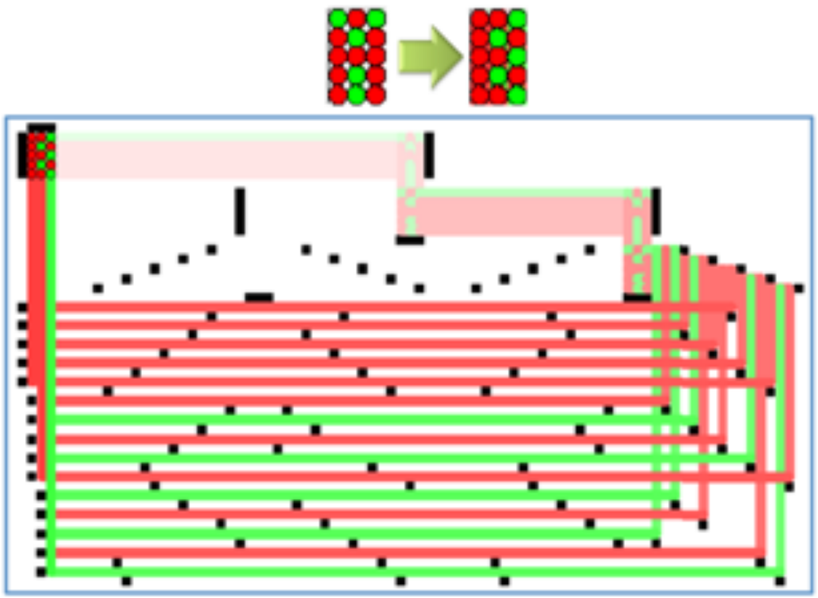


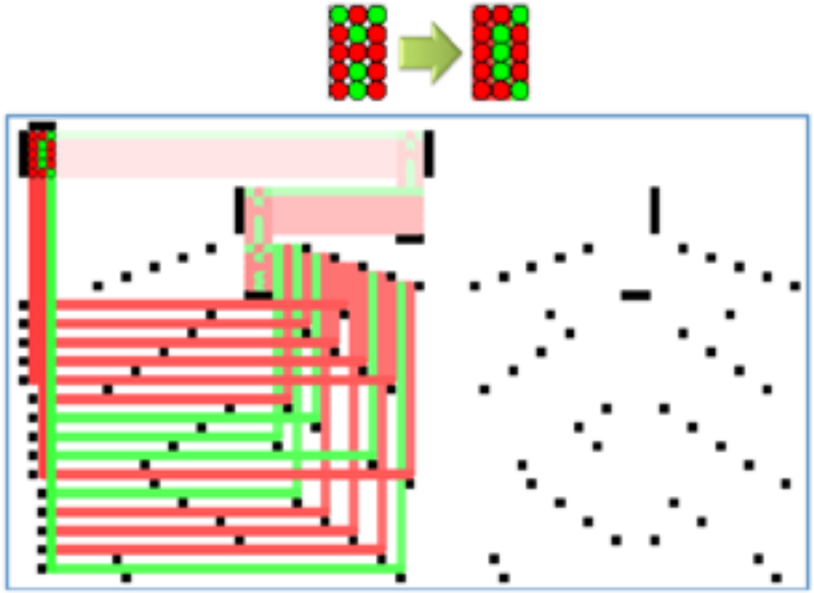
How Obstacles Can Be Helpful



Multiple Permutations

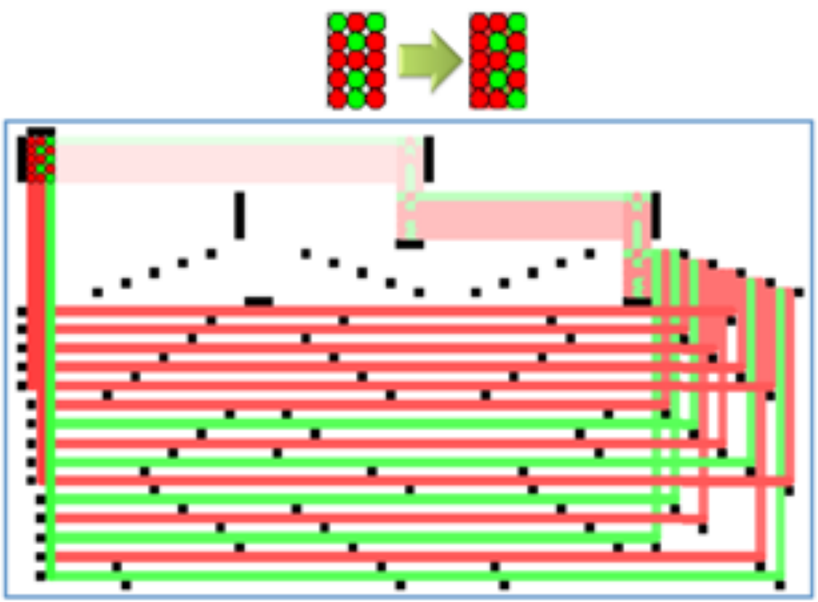


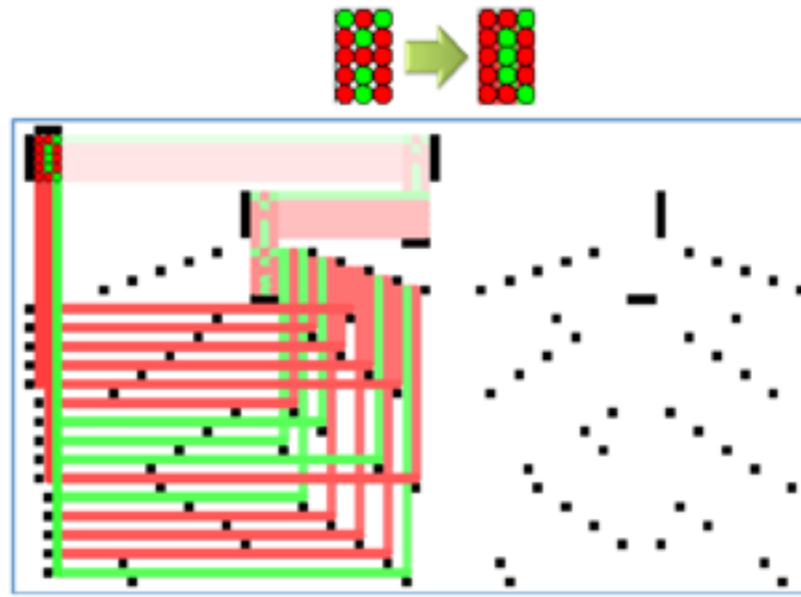




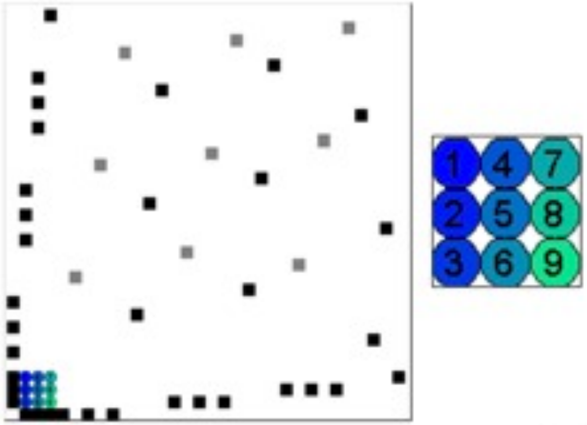
Multiple Permutations

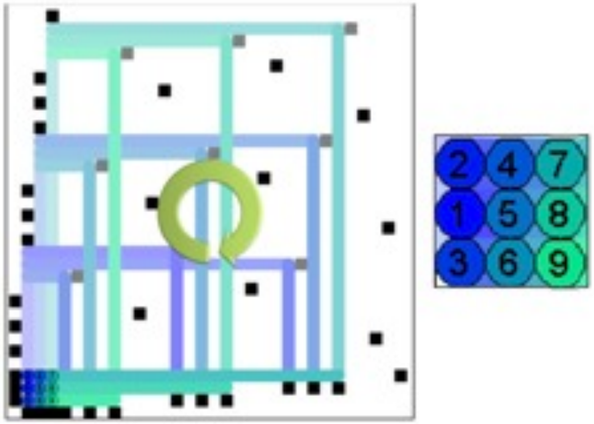
Theorem 3. *For any set of k fixed, but arbitrary, permutations of $n \times n$ pixels, we can construct a set of $O(kN)$ obstacles, such that we can switch from a start arrangement into any of the k permutations using at most $O(\log k)$ force-field moves.*

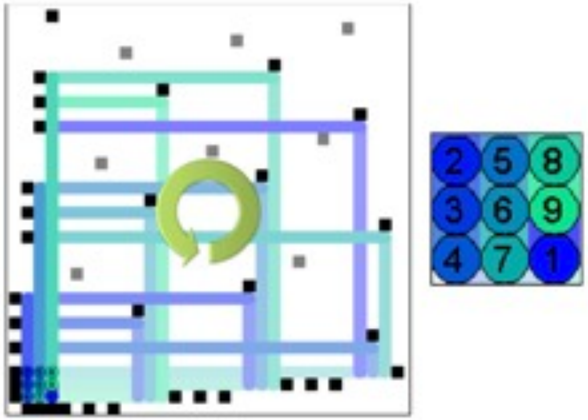




Designing Obstacles







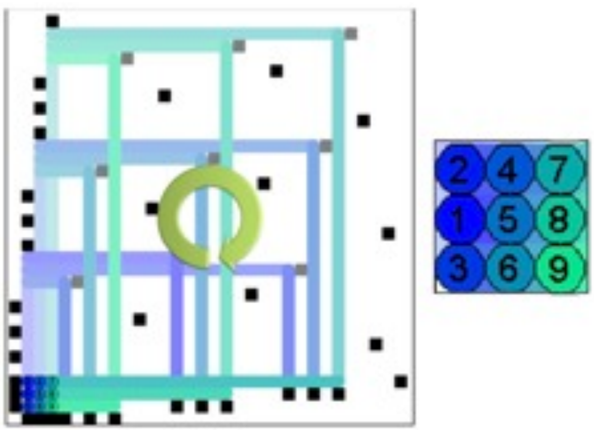
CW: (12)

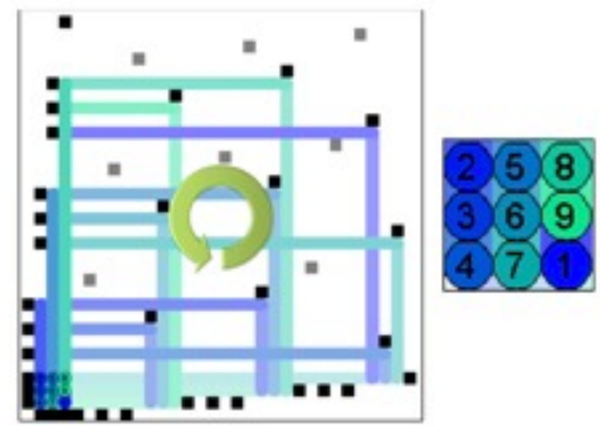
CCW: (123456789)

Designing Obstacles

Lemma 5. *Any permutation of N objects can be generated by the two base permutations $p = (12)$ and $q = (12 \cdots N)$. Moreover, any permutation can be generated by a sequence of length at most N^2 that consists of p and q.*

Theorem 6. *We can construct a set of $O(N)$ obstacles such that any $n \times n$ arrangement of N pixels can be rearranged into any other $n \times n$ arrangement π of the same pixels, using at most $O(N^2)$ force-field moves.*





CW: (12)

CCW: (123456789)

Designing Obstacles

Lemma 7. *Any permutation of N objects can be generated by the N base permutations $p_1 = (12), p_2 = (13), \ldots, p_{N-1} = (1(N-1))$ and $q = (12 \cdots N)$. Moreover, any permutation can be generated by a sequence of length at most N that consists of the p_i and q.*

Theorem 8. *We can construct a set of $O(N^2)$ obstacles such that any $n \times n$ arrangement of N pixels can be rearranged into any other $n \times n$ arrangement π of the same pixels, using at most $O(N \log N)$ force-field moves.*

Theorem 9. *Suppose we have a set of obstacles such that any permutation of an $n \times n$ arrangement of pixels can be achieved by at most M force-field moves. Then M is at least $\Omega(N \log N)$.*

Proof. Each permutation must be achieved by a sequence of force-field moves. Because each decision for a force-field move $\{u, d, l, r\}$ partitions the remaining set of possible permutations into at most four different subsets, we need at least $\Omega(\log(N!)) = \Omega(N \log N)$ such moves. □

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North-Holland

THE COMPLEXITY OF FINDING MINIMUM-LENGTH GENERATOR SEQUENCES

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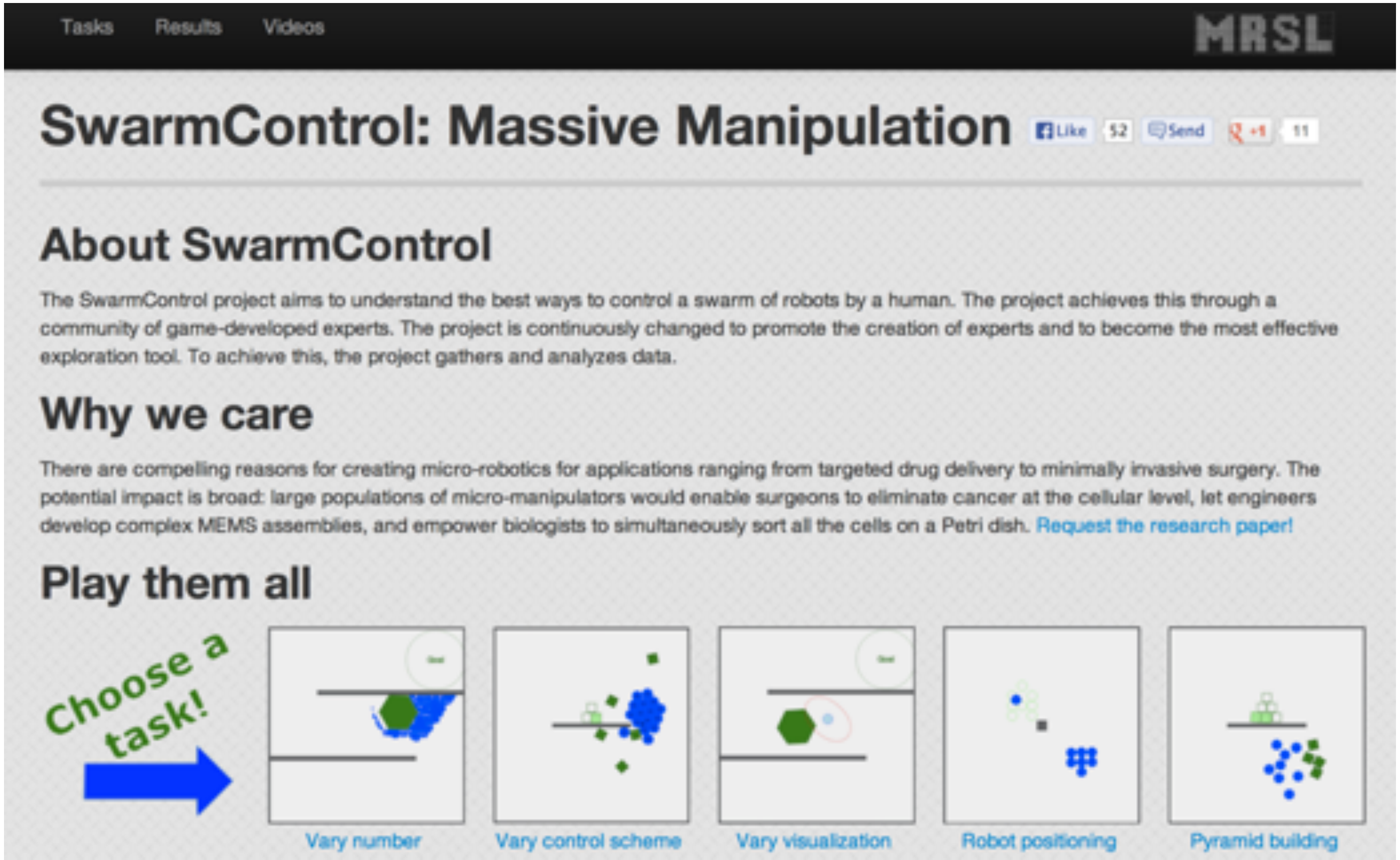
Revised May 1984

Abstract. The computational complexity of the following problem is investigated: Given a permutation group specified as a set of generators, and a single target permutation which is a member of the group, what is the shortest expression for the target permutation in terms of the generators? The general problem is demonstrated to be PSPACE-complete and, indeed, is shown to remain so even when the generator set is restricted to contain only two permutations. The restriction on generator set cardinality is the best possible, as the problem becomes soluble in polynomial time if the generator set contains only one permutation. An interesting feature of this problem is that it does not fall under the headings of 'two person games' or 'formal languages' which cover the great majority of known PSPACE-complete problems. Some restricted versions of the problem, in which the generator set is fixed rather than being part of the problem instance, are also

Conclusions

- More work in theory and practice!
- Work in progress
- Other constraints and objectives
- 3D
- More general motion
- Practical constraints
- Heterogeneous objects
- Objects that bond

Sales Pitch



http://www.swarmcontrol.net

Thank you!